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Invisible Matter

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Abstract

These lectures have been given to particle physicists, mostly experimentalists and very briefly and at a pedestrian level review the problems of dark matter. The content of the lectures is the following: 1. Introduction. 2. Cosmological background. 3. Luminous matter. 4. Primordial nucleosynthesis and the total amount of baryons. 5. Gravitating invisible matter. 6. Baryonic crisis. 7. Inflationary omega. 8. Intermediate summary. 9. Possible forms of dark matter. 10. Structure formation: basic assumptions. 11. Structure formations: basics of the theory. 12. Evolution of perturbations with different forms of dark matter. 13. Conclusion. The presentation and conclusion reflects personal view of the author that a considerable amount of invisible energy in the universe is in the form of vacuum energy (cosmological constant) and possibly in the form of a classical field which adjusts vacuum energy to the value permitted and requested by astronomical data.

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1 Introduction

There are very strong indications that the bulk of matter in the universe is not the normal electron-nucleon stuff but something unknown which may be made of not yet discovered elementary particles, or of some field excitations at astronomically large scales which are stable by topological reasons, or maybe even something else. This unknown form of matter contributes more than 90% into total mass (or better to say, energy) density in the universe. This matter is usually called dark matter. The name reflects the fact that it is not luminous and maybe also the level of our knowledge of the subject. A better name is invisible matter since it neither emits nor absorbs light. The only observed manifestations of this matter are gravitational effects. There are also some theoretical arguments in favor of existence of dark matter but, though very persuasive, they are not absolutely compelling. It is the biggest challenge in physics of this and may be of the next century to directly observe this new (if it is indeed new) form of matter and to study its properties.

In these lectures I will address the following questions:

1. Is there indeed dark (invisible) matter and why do we think that it is so?
2. Could all dark matter be baryonic? If not all, could there still be some invisible baryons?
3. If not baryonic, what is it? There are two ways to address this question: one is to use the elementary particle theory for possible candidates for the constituents of dark matter and the other is to use the theory of large scale structure formation in the universe and the astronomical observations to constraint the properties of dark matter particles.

In the next Section the necessary cosmological background is briefly presented. In sec. 3 the luminous matter is discussed. Nucleosynthesis constraints on the amount of baryons in the universe are presented in Sec. 4. Astronomical data indicating gravitational action of

invisible matter are considered in Sec. 5. In sec. 6 the so-called baryonic crisis created by a too large an amount of hot gas observed in rich galactic clusters is discussed. Prediction of inflationary cosmology on the value of Ω is given in sec. 7. In sec. 8 the summary of measurements of Ω for different forms of matter is presented. A brief discussion of possible cosmic relics which may exist in the universe and contribute to the dark matter is given in sec. 9. In sec. 10 a critical analysis of the basic assumptions of the theory of large scale structure formation is presented. A brief introduction to the basics of the theory of structure formation is given in sec. 11. Evolution of density perturbations in the universe dominated by different forms of dark matter is described in sec. 12. In Conclusion some speculations on the best bet about the form of dark matter is presented and possible experiments and theoretical problems, which may shed light on the nature of dark matter, are discussed. I will not review the existing direct experimental searches of dark matter particles in low background experiments. It is done by L.Mosca in this School.

2 Cosmological Background

The source of gravitational field is the energy-momentum tensor $T_{\mu\nu}$. In homogeneous cosmological models it is presented in the ideal liquid form which in the rest frame of matter can be written as:

$$T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p) \quad (1)$$

where ρ and p are respectively the energy and pressure density. It is usually assumed that at a later stage the universe is dominated by nonrelativistic matter when p may be neglected and the energy density of nonrelativistic matter, $\rho = \rho_m$, is practically equal to the mass density. Indeed as a function of the scale factor $a(t)$, which describes the universe expansion, the energy density of nonrelativistic matter goes down as $\rho_m \sim a^{-3}$, while that of relativistic one as $\rho_{rel} \sim a^{-4}$. The late dominance of nonrelativistic matter may be not true in models with long-lived decaying particles producing relativistic species but in the standard cosmology it is

fulfilled with a very good precision. Apart from contributions to $T_{\mu\nu}$ from normal relativistic or nonrelativistic matter there may be also a rather mysterious contribution from vacuum energy with $T_{\mu\nu} = \rho_{vac}g_{\mu\nu}$ where $g_{\mu\nu}$ is the metric tensor. In contrast to the energy density of matter ρ_{vac} stays constant in the course of the universe evolution. Not long ago a nonzero ρ_{vac} was considered as rather exotic by majority of astronomers and was usually neglected but now with accumulation of new data the attitude is changing.

Roughly speaking the matter in the universe can be divided into three categories: 1) Visible or luminous matter which is observed either by emitted or absorbed light. Almost surely this is the normal baryonic stuff. 2) Invisible normal baryonic matter. We can make a conclusion about its amount considering primordial nucleosynthesis. 3) Invisible nonbaryonic matter. This one is observed only by its gravitational action. It can also be divided according to the spatial distribution: clustered or uniform. We can "feel" the former by observing the motion of the visible tracers like gas around galaxies or galactic satellites while the latter can be observed only by cosmological effects and hence the observations are more complicated and less secure. In particular vacuum energy belongs to the last case.

The magnitude of mass or energy density in the universe, ρ , is usually presented in terms of the dimensionless parameter

$$\Omega = \rho/\rho_c \quad (2)$$

where ρ_c is the so called critical or closure density,

$$\rho_c = \frac{3H^2 m_{Pl}^2}{8\pi} = 1.88 h_{100}^2 \times 10^{-29} g/cm^3 = 10.5 h_{100}^2 KeV/cm^3 \quad (3)$$

Here $m_{Pl} = 1.22 \times 10^{19} GeV$ is the Planck mass (the Newtonian gravitational constant is $G_N = m_{Pl}^{-2}$) and h_{100} is the dimensionless Hubble parameter,

$$H = 100 h_{100} km/sec/Mpc \quad (4)$$

The value of H is very poorly known, $0.4 < h_{100} < 1$. The universe age, as we see below, favors lower values of H but the recent data indicate that $h_{100} = 0.7 - 0.8$ or maybe even higher[1].

Integrating the Einstein equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\rho}{3m_{Pl}^2} - \frac{k}{a^2}, \quad (5)$$

which governs the expansion of the universe, one can express the universe age through the present day value of the Hubble constant and the fractions of energy density of different forms of matter (for more detail see e.g. my lecture in the previous ITEP school [2] or the textbooks [3, 4, 5, 6]):

$$t_u = \frac{1}{H} \int_0^1 \frac{dx}{\sqrt{1 - \Omega_{tot} + \Omega_m x^{-1} + \Omega_{rel} x^{-2} + \Omega_{vac} x^2}} \quad (6)$$

where Ω_m , Ω_{rel} , and Ω_{vac} correspond respectively to the energy density of nonrelativistic matter, relativistic matter, and to the vacuum energy density (or, what is the same, to the cosmological constant); $\Omega_{tot} = \Omega_m + \Omega_{rel} + \Omega_{vac}$, and $1/H = 9.8h_{100}^{-1} \times 10^9 \text{yr}$.

The universe age is estimated by two different methods: by nuclear chronology which uses measurements of the ratios of the long-lived isotopes, $^{187}\text{Re}/^{232}\text{Th}$ or $^{238}\text{U}/^{235}\text{U}$, and by the estimated age of old stellar clusters. Both methods give the age in the range of (12 - 20) Gyr. The recent analysis gives $t_u \approx 18$ Gyr (for the review and the list of references see e.g. paper [7]).

If the universe is dominated by nonrelativistic matter, as is usually assumed, i.e. $\Omega_{tot} = \Omega_m$, then the universe age is approximately given by the expression $t_u \approx 9.8h_{100}^{-1}/(1 + \sqrt{\Omega_m}/2)$ Gyr. Even with $\Omega = 0$ the big universe age and large H are inconsistent. The inconsistency becomes stronger for $\Omega_{tot} = 1$ predicted by inflationary models. The observed tendency to large values of H and t_u presents a very strong argument in favor of nonvanishing vacuum energy. For $\Omega_{tot} = \Omega_m + \Omega_{vac} = 1$ eq.(6) gives

$$t_u = \frac{2}{3H\sqrt{\Omega_{vac}}} \ln \frac{1 + \sqrt{\Omega_{vac}}}{\sqrt{1 - \Omega_{vac}}} \quad (7)$$

To get, let us say, $t_u = 14$ Gyr for $h_{100} = 0.75$ we need $\Omega_{vac} \approx 0.8$. If this is true the bulk of energy in the universe is just the energy of empty space, of vacuum. Unfortunately

our understanding of vacuum energy is very poor. Any reasonable estimate of it gives the result which is some 50-100 orders of magnitude higher than the observational limit $\rho_{vac} < 10^{-47} GeV^4$ (for the review see refs. [8, 9]). For example there are contributions from quark and gluon condensates into ρ_{vac} which are well established in QCD and which are about $10^{-4} GeV^4$. So there must exist a contribution into vacuum energy from something not related to quarks and gluons but with exactly the same magnitude and the opposite sign. It is hard to imagine an accidental cancellation with such an accuracy but no dynamical mechanism has yet been found. If something is very small, one would naturally expect this to be exactly zero and this is a general attitude to vacuum energy. This is one of the reasons why models with a nonzero cosmological constant were not seriously considered by the establishment. Another reason for that is an unnatural coincidence of the present-day values of vacuum energy and the critical energy density of the universe. As we mentioned above they are pretty close to each other. However the critical energy density is decreasing with the universe age as $1/t^2$ while ρ_{vac} remains constant (at least in the standard model). The new astronomical observations changed the attitude to cosmological constant and now it is considered more seriously. Moreover, as we see below, models with nonzero vacuum energy have some advantages in the description of the structure formation in the universe. To my mind the best possibility in solving the cosmological constant problem is the adjustment mechanism [10, 11] which ensures the cancellation of vacuum energy by an action of a new field coupled to gravity or curvature so that its condensate cancels out any initial ρ_{vac} . This cancellation is generically not complete and a noncompensated amount of ρ_{vac} is always of the order of ρ_c . In such cosmological models both vacuum energy and the energy of the new field are essential at any stage of the universe evolution (in contrast to the models with normal time independent vacuum energy) and this might have an impact on primordial nucleosynthesis, structure formation, etc. The progress here is inhibited by an absence of a consistent (even toy) model.

3 Luminous Matter

The amount of luminous matter in the universe is estimated in the following way. The flux of light, F , from different sources is measured on the Earth and the luminosity of an individual source is found by the relation:

$$L_i = 4\pi F_i l_i^2 \quad (8)$$

where l_i is the distance to the source. Then one may estimate the average mass density of luminous matter as $\rho_{lum} = \sum_i L_i (M_i/L_i)/V$ where V is a (large) volume over which the averaging is done. Usually the value $M/L = 5M_\odot/L_\odot$, which is typical for the stellar (galactic) matter, is substituted for the ratio M_i/L_i (here $M_\odot = 2 \times 10^{33} g$ and $L_\odot = 4 \times 10^{33} erg/sec$ are respectively the solar mass and luminosity). The measured flux permits to find the luminosity density as (see e.g. ref. [12]):

$$\lambda = \sum_i L_i/V = (2 \pm 0.6) \times 10^8 h_{100} L_\odot Mpc^{-3} \quad (9)$$

where $1 Mpc = 3.1 \times 10^{24} cm$. The result is proportional to the Hubble constant because the distance l is determined by the Hubble flow, $v = Hl$ and correspondingly $l^2/V \sim h_{100}$. The mass density of luminous matter is given by $\rho_{lum} = \lambda(M/L) = 5(\lambda/L_\odot)M_\odot = (0.7 \pm 0.2) \times 10^{-31} h_{100} g/cm^3$.

Using these numbers we find for the relative amount of the luminous matter:

$$\Omega_{lum} = \rho_{lum}/\rho_c = (0.35 \pm 0.10)\% h_{100}^{-1} \quad (10)$$

This is by far smaller than the amount of nonluminous matter.

4 Primordial Nucleosynthesis and the Net Amount of Baryons

Though some (or maybe even the bulk) of baryons are invisible there is a way to find their total number density with a relatively good precision. This is based on consideration of

primordial nucleosynthesis of light elements such as deuterium, helium-3, helium-4, and lithium-7. Their calculated abundance are sensitive to the cosmic baryon number density or, better to say, to the baryon-to-photon ratio, $\eta_{10} = 10^{10} n_B / n_\gamma = 10^{10} \eta$. The starting point of the nucleosynthesis is the fixation of the proton-to-neutron ratio. It is determined by the competition between the weak interaction reactions, $pe^- \leftrightarrow n\nu_e$ and $p\bar{\nu}_e \leftrightarrow ne^+$, and the universe expansion rate. To the moment when these reactions become ineffective the (n/p)-ratio freezes approximately at 1/6. It takes place roughly at $T = 0.65 \text{ MeV}$. Later on at $T \approx 0.065 \text{ MeV}$, when the formation of light nuclei begins, this ratio drops down to 1/7 because of neutron decay. The magnitude of n/p -ratio does not depend upon η because the corresponding reactions are linear with respect to baryons and so their rate, \dot{N}/N , is independent of the baryon number density. Note that n/p -ratio is rather sensitive to the number of different particle species in the primeval plasma and this permits in particular to put a bound on the number of different massless ($m < 1 \text{ MeV}$) neutrino species. For the up-to-date analysis of this bound see refs.[14, 15, 16].

Since $\eta \approx 3 \times 10^{-10}$ is a very small number, the amount of the produced nuclei is tiny even at the temperatures which are smaller than their binding energy. For example the number density of deuterons is determined by chemical equilibrium and is equal to

$$n_D = n_n n_p e^{B_D/T} \left(\frac{m_D T}{2\pi} \right)^{3/2} \left(\frac{2\pi}{m_N} \right)^3 \quad (11)$$

where $B_D = 2.224 \text{ MeV}$ is the deuterium binding energy. One can see that n_D becomes comparable to n_n at the temperature:

$$T_D = \frac{0.065 \text{ MeV}}{1 - 0.03 \ln \eta_{10}} \quad (12)$$

Above this temperature the number density of deuterons in the cosmic plasma is negligible and correspondingly the formation of other nuclei, which proceeds through collisions with deuterium is suppressed. This is the so called "deuterium bottleneck". But as soon as T_D is reached, the nucleosynthesis goes very quickly and practically all the neutrons, which existed

in the cosmic plasma at that time, are captured into helium-4. The latter has the largest binding energy, $B_{4He} = 28.3 \text{ MeV}$ and so in equilibrium its abundance should be the largest. Its mass fraction, $Y(^4He)$, is determined predominantly by the (n/p) -ratio and is equal to $2(n/p)/[1 + (n/p)] \approx 25\%$.

The time moment corresponding to $T = T_D$ is determined by the well known relation:

$$t_D = 0.16 \left(\frac{3.37}{g_*} \right)^{1/2} \left(\frac{m_{Pl}}{T^2} \right) = 310 \text{ sec} \left(\frac{3.37}{g_*} \right)^{1/2} (1 - 0.03 \ln \eta_{10})^2 \quad (13)$$

This relation is obtained by equating the expression for the critical energy density of relativistic matter

$$\rho_c = 3m_{Pl}^2/32\pi t^2 \quad (14)$$

(see eq.(3) with $H = 1/2t$) and the energy density of equilibrium relativistic plasma with temperature T

$$\rho_T = \pi^2 g_* T^4/30 \quad (15)$$

Here g_* is the number of relativistic species in the plasma. For a photon and three types of neutrinos with equal temperature $g_* = 7.25$. After electron-positron annihilation the temperature of photons became 1.4 larger than that of neutrinos and $g_* = 3.37$. One sees that with a larger η the time t_D is getting smaller and the number of not-yet-decayed neutrons is larger, $n_n \sim \exp(-t_D/\tau_n)$, where $\tau_n = 890 \text{ sec}$. Thus $Y(^4He)$ is a rising function of η and its variation is relatively weak, with η_{10} changing from 1 to 10 the mass fraction of helium changes by approximately 10%. Still since $Y(^4He)$ is rather accurately known it can be used as a serious indicator of η and correspondingly of the **total** baryonic mass density.

In contrast to 4He the fraction of deuterium is a strong and a decreasing function of η . Physically it is easy to understand: more baryons are in the plasma, more efficient are the processes of disappearance of deuterium through formation of heavier nuclei with larger binding energies, like e.g. $D + p \rightarrow ^3He + \gamma$ and $^3He + n \rightarrow ^4He + \gamma$. The destruction of

deuterium in the first process is governed by the equation:

$$\dot{n}_D = -\sigma v n_D n_p \quad (16)$$

where σ is the cross-section of the reaction and v is the velocity of the colliding particles. We assumed that the temperature is low enough so that the inverse process is Boltzmann suppressed and is not effective.

Integrating this equation under assumption that $\eta_p = n_p/n_\gamma \approx \eta = \text{const}$ we find:

$$n_D = n_{Di} \exp[-0.06\eta(3.77/g_*)^{1/2}\sigma v m_{Pl} T_i] \quad (17)$$

where n_{Di} and T_i are the initial values of the number density and temperature which should be taken at the moment when the production of deuterium by photo-dissociation of heavier nuclei effectively stops. Eq. (17) illustrates the made above statement about the dependence of deuterium abundance on η . ${}^3\text{He}$ has the similar dependence on η but somewhat less strong. The net output of these elements is much smaller than that of ${}^4\text{He}$, $D/H \approx 10^{-4}$ and ${}^3\text{He}/H \approx (1-2) \times 10^{-5}$ by number.

The fraction of ${}^7\text{Li}$ depends on η nonmonotonically having a minimum at $\eta_{10} \approx 3$. This more complicated dependence is connected with a competition of different production channels.

Though the abundance of D , ${}^3\text{He}$, and ${}^7\text{Li}$ are not so well known as that of ${}^4\text{He}$, their strong dependence on η also permits to use them as indicators of the total baryonic mass density in the universe.

Of course the calculations of the light element abundance are not done in this naive way. We use it here only for the qualitative description of the phenomenon. The calculations are based on the numerical integration of the complete set of kinetic equations with the experimentally measured nuclear cross-sections. With the fixed values of the parameters the theoretical results are very accurate but the direct comparison with observation is difficult not only because of the observational uncertainties but also because of complicated evolutionary

effects. Due to the latter the light element abundance in the present-day universe may be quite different from the primordial ones. It is relatively simple with ${}^4\text{He}$ because it can only be produced but not destroyed in the course of the universe evolution. The production of helium-4 in stars is accompanied by the production of oxygen, nitrogen, and carbon (they are called metals by astronomers). Because of that the observations of primordial helium are done in the regions of the sky least contaminated by these heavier elements. Moreover the data on helium-4 and the heavy elements are linearly extrapolated to the so called zero-metallicity state. In this way a rather safe upper bound on $Y({}^4\text{He})$ can be obtained.

The evolutionary behavior of other light elements is more complicated, they can be both destroyed and produced in stellar processes and the comparison of the theory with observations is difficult. Still only two years ago the agreement between the theory and observations was believed to be excellent. In a sense this is true because the theory predicts the abundance of different light elements which span the region between 25% for ${}^4\text{He}$ down to 10^{-10} for ${}^7\text{Li}$ and this is definitely confirmed by the data. The problem appeared at the level of more refined conclusions. The old data for all elements: ${}^4\text{He}$, D , ${}^3\text{He}$, and ${}^7\text{Li}$, were in agreement with the theory for the common value of $\eta_{10} = 3.5 \pm 1$. The best fit to the data was obtained with the three known massless neutrinos which was also an argument in favor of the overall agreement. With this value of η the baryonic mass fraction in the universe is

$$\Omega_B = 4 \times 10^{-3} \eta_{10} h_{100}^{-2} = (1.4 \pm 0.4)\% h_{100}^{-2} \quad (18)$$

In this case $\Omega_B/\Omega_{lum} = (2 - 7)h_{100}^{-1}$ and the baryons in the universe are mostly invisible.

A new measurement of deuterium abundance [17, 18] at high redshift $z = 2.9$ showed a surprisingly high value, $D/H = (1.9 - 2.5) \times 10^{-4}$. It was done by the measurement of absorption of light from a distant quasar in a Lyman alpha cloud. The deuterium in this cloud did not suffer from evolution and so its abundance should be equal to the primordial one. This value is an order of magnitude bigger than the value found in interstellar medium. This would request η approximately three times smaller than the previously accepted and,

if so, practically all baryons in the universe are visible. The analysis of the new data on helium-4 [15] also suggests a smaller η . However another group [19] reported much smaller value, $D/H = (1-2) \times 10^{-5}$. So the problem remains unsettled. A smaller value of η creates problems with the abundance of ${}^3\text{He}$ which in this case should be considerably larger than that observed in our neighbourhood. For the analysis of the new data and evolutionary effects one can address the recent papers [15, 14, 20]. In accordance with the analysis of helium-4 made in ref. [15] the best value for the number of neutrino species is $N_\nu = 2.2 \pm 0.27 \pm 0.42$ if the old value $\eta_{10} = 3.6$ is assumed. Though the central value of N_ν is below three the error bars are generous enough to be consistent with the standard three neutrinos.

So we have two interesting possibilities which can be resolved with an improved accuracy. The first is that η is close to the old value so that there are invisible baryons and $N_\nu < 3$. It may mean in particular that ν_τ is heavy and unstable. Another more conservative possibility would be realized if D/H is high, η is small and thus all baryons are visible, and $N_\nu = 3$. Small η may imply a problem with the observed ${}^3\text{He}$ but evolutionary effects could be significant (see ref. [15]). A large universe age looks favorable from the point of view of primordial deuterium destruction. This can explain a large amount of primordial D/H observed in Lyman alpha clouds and a small amount of it in interstellar medium nearby. The large t_U in turn is possible only with nonzero vacuum energy. If an adjustment mechanism for its cancellation is operating then the noncompensated amount was essential during all stages of the universe evolution and this might have an unknown impact on the primordial nucleosynthesis. However if we forget about this rather uncertain possibility, the conclusion that the primordial nucleosynthesis does not permit the mass fraction of baryons to be larger than a few per cent seems to be sound enough.

5 Gravitating Invisible Matter

The observation that there is more gravitating matter in the universe than is directly seen was made more than half a century ago by Oort [21] for the Galaxy and by Zwicky [22] for Coma cluster. Virial considerations showed that the visible matter alone cannot account for the observed velocities. To the present day a very rich data were accumulated which strongly support the conjecture that there is a large amount of invisible matter in the universe. Velocities of HI gas clouds around galaxies are measured by 21 cm line for hundreds of galaxies up to distances $r_{max} = 30 \text{ Kpc}$. All this curves becomes flat, $v \rightarrow \text{const}$, for r outside the luminous centre, $r > r_{gal} = 10 \text{ Kpc}$. The equilibrium velocities are determined by the relation

$$\frac{v^2}{r} = \frac{G_N M(r)}{r^2} \quad (19)$$

where $G_N = m_{Pl}^{-2}$ is the Newtonian gravitational constant and $M(r)$ is the mass inside radius r . This equation implies that for the mass confined inside r_{gal} and for $r > r_{gal}$ the velocity goes down with the radius as $1/\sqrt{r}$, while for v tending to a constant, $M(r) \sim r$. Since r_{max} is considerably larger than the luminous galactic radius the amount of invisible matter in galaxies should be much larger than that of the visible one.

The law $M(r) \sim r$ implies that the mass density $\rho(r)$ falls down as $1/r^2$. This takes place for the isothermal selfgravitating gas as follows from the equations:

$$\Delta\phi = 4\pi G_N \rho \quad (20)$$

and

$$\rho = \rho_0 \exp(-\phi/T) \quad (21)$$

There is a very important question how far, that is to what maximum value of r , the law $M(r) \sim r$ or $v = \text{const}$ is valid. The measurement of HI gas velocities which can be done up to 30 Kpc does not show any cutoff and for large spirals give $M_{DM}/M_{lum} = 3 - 5$. Measurements of velocities of satellite galaxies around the Milky Way and other large spirals

can be extended up to 200 Kpc and give $M_{DM}/M_{lum} = 12-15$. This gives $\Omega_{spirals} \approx 0.09h_{100}^{-1}$. Whether dark halos go beyond this distance is unknown. No cut-off is observed with all existing observations. Elliptical galaxies where dark matter is traced up to 100 Kpc by hot X-ray gas give $M_{DM}/M_{lum} \geq 10$. An last but not the least (though less secure) the clusters of galaxies give $M_{DM}/M_{lum} = 50 - 100$.

The determinations of the gravitating masses by rotational velocities, considered above, are valid for gravitationally bound systems and applicable to masses at relatively small scales, at most of cluster of galaxies. There is another way to determine gravitating masses which can be applied to considerably larger scales. It is based on the measurements of the large scale flows. The latter are the so called peculiar velocities i.e. the velocities of different galaxies with respect to the general Hubble expansion. By assumption these velocities are induced by the density contrast, $\delta = (\rho - \rho_0)/\rho_0$, where ρ_0 is the homogeneous cosmological energy density. In other words peculiar velocities are proportional to the gravitational acceleration created by the contrast of the gravitational potential, $\delta\phi$. The theory of this phenomenon is simplified because at large scales $\delta \ll 1$ and the linear approximation is valid.

From the continuity equation, $\partial_t \rho + \vec{\nabla}(\rho \vec{v}) = 0$, one can find in the linear regime:

$$\vec{\nabla} \vec{v} = -\partial_t \delta \quad (22)$$

where $a(t)$ is the cosmological scale factor.

For sufficiently large wave lengths (larger than the Jeans wave length, see below sec. 11) the behaviour of δ at matter dominated stage is governed by the equation (see e.g. [23, 24]):

$$\ddot{\delta} + 2(\dot{a}/a)\dot{\delta} - 4\pi G_N \Omega \rho_0 \delta = 0 \quad (23)$$

This equation can be easily solved in two limiting cases: 1) $\Omega = 1$ for which the rising (gravitationally unstable) mode goes as $\delta \sim a(t) \sim t^{2/3}$ and 2) $\Omega = 0$ for which $\delta = const$. In the intermediate case, $0 < \Omega < 1$, the solution can be approximately presented as [23]:

$$(\dot{\delta}/\delta) = \Omega^{0.6}(\dot{a}/a) \equiv H\Omega^{0.6} \quad (24)$$

So from eq. (22) follows

$$\vec{\nabla} \vec{v} = -H\Omega^{0.6} \left(\frac{\delta\rho}{\rho} \right)_{tot} = -\frac{H}{b}\Omega^{0.6} \left(\frac{\delta\rho}{\rho} \right)_{vis} \quad (25)$$

where the biasing factor b is introduced to relate the visible density contrast to the total one. Essential assumptions which are made in the derivation of this equation are the linearity i.e $\delta_{tot} = \delta_{vis}/b$ and the scale independence of this relation. It is also assumed that the visible density contrast is given by the excess in galaxy number count $\delta n/n$. All these assumptions may be incorrect to a larger or smaller extent.

It is noteworthy that this equation does not contain the ambiguity connected with the cosmological distance scale or (what is the same) with the value of the Hubble constant H . Indeed the integration of the equation gives $\int H dr \sim v_H$ where v_H is the velocity of the Hubble expansion and is directly measurable. Note also that our derivation of eq.(22) is oversimplified and can be used for illustrative purposes only; for a more rigorous approach see e.g. refs.[5, 23].

The velocities of galaxies are measured by the red-shift and so only the component in the direction to the observer can be found. This extra problem can be solved if the velocity field is rotationless, $\vec{\nabla} \times \vec{v} = 0$ [25]. It is true for pure gravitational interactions for which an initially curlless velocity field remains curlless. Thus the velocity field can be expressed through a single scalar function, velocity potential, and this permits to determine $\vec{\nabla} \vec{v}$ from observations and to find Ω from eq. (25). The recent analysis [26] of peculiar velocities of about 3000 galaxies gives

$$\Omega^{0.6}/b = 0.74 \pm 0.13 \quad (26)$$

So if $\Omega = 1$ then $b = 1.35 \pm 0.23$ or if $b = 1$ then $\Omega = 0.61 \pm 0.18$. If $\Omega < 0.3$ one has to introduce antibiasing, $b < 1$. Though it is not absolutely excluded, the physics of that is rather unclear. For the review of this method and possible errors one can address to the paper [27]. It is argued that the lower bound on Ω independent of biasing is $\Omega > 0.3$. This conclusion however is based on the assumption of vanishing cosmological constant. It is

worthwhile to redo the analysis without this assumption. Some other potential reasons that may invalidate the above conclusion are a possible dependence of b on scale and a relatively small size of the analyzed sample of galaxies. Another reason which could also lead to the violation of the statement about large Ω is a possible nongravitational contribution to the large scale flow. If for example the large observed dipole anisotropy of the cosmic microwave background (cmb) is not induced by the gravitational acceleration but has its own (intrinsic) origin like isothermality fluctuations with very large (superhorizon) wave length, the results based on the measurement of the large scale flow may be invalid. Typically this kind of mechanism would create the quadrupole q and higher multipole anisotropy of cmb of the same order as the dipole (d) while observationally $d \approx 2 \times 10^{-3}$ and $q = (afew) \times 10^{-6}$. This problem is analyzed in the recent paper [28] where it is argued that it is still possible to get a large dipole while all other multipoles would be small. Note that the model of baryon island universe [29], where the baryon asymmetry was generated only inside a spherical bubble with the present-day size corresponding to $z = 5 - 10$, provides $q \approx d^2$ and very small other multipoles. The dipole and other asymmetries in this model are connected with the shift in our position with respect to the geometrical center of the island. The smallness of the dipole means that we live almost in the Center of the Universe. This is not very natural but rather appealing with regard of our vanity.

6 Baryonic Crisis

Recent observations [30, 31] showed a surprisingly large fraction of baryons in rich clusters of galaxies. In the first paper [30], which drew attention to the problem, the mass of the hot gas in the Coma cluster was estimated by the intensity of X-ray emission measured by Rossat satellite, $M_{gas} = (5.45 \pm 1) \times 10^{13} h_{100}^{-5/2} M_{\odot}$. It dominates the mass of matter contained in the stars, $M_{star} = (1 \pm 0.2) \times 10^{13} h_{100}^{-1} M_{\odot}$. The total mass of the cluster was found by two methods: by the virial consideration and by the condition of the thermal hydrostatic

equilibrium in the gas; both ways give the close results: $M_{total} = (6.7 \pm 1) \times 10^{14} h_{100}^{-1} M_{\odot}$. Thus the ratio of the mass contained in baryons to the total mass of the cluster is

$$r_B \equiv \frac{M_{bar}}{M_{tot}} = (0.09 \pm 0.05) h_{100}^{-3/2} \quad (27)$$

Similar results have been obtained in ref.[31] from the analysis of 5 clusters (Coma plus four others):

$$r_B > \frac{M_{gas}}{M_{tot}} = (0.03 - 0.08) h_{100}^{-3/2} \quad (28)$$

As has been argued in ref. [32] these results give safe lower bounds on r_B and the real numbers should be noticeably higher.

Now if we assume that the mass contained in the clusters represents a fair sample of the total mass in the universe then these data together with the nucleosynthesis constraint on Ω_B put the strong upper bound on the energy density in the universe:

$$\Omega_{cluster} < 0.05 \eta_{10} h_{100}^{-1/2} (0.1/r_B) \quad (29)$$

This bound is evidently valid for the clustered matter and is not applicable for the uniformly distributed one.

The authors of ref. [30] have discussed several different explanations of the phenomenon like nonstandard nucleosynthesis, nongravitational processes in the structure formation, possible mistakes in the standard technique in the estimation of the cluster mass, and an open universe model with $\rho_{tot} \ll \rho_c$. The last possibility is disfavored by the standard inflationary scenario (see below). To my mind these data give a good indication to nonzero vacuum energy, $\Omega_{vac} \approx 0.8$. It is uniformly distributed and for the typical cluster size about 1 Mpc does not contribute much inside this radius. The nonzero value of vacuum energy is rather natural by the reasons mentioned above. Another evident possibility is a large contribution from the so called hot dark matter which may be also nonclustered at these scales. This however seems to be excluded because the total amount of hot dark matter in the universe cannot exceed 30% at least in the models with scale-free spectrum of initial perturbations (see below Sec. 12)

7 Inflationary Omega

Aesthetically $\Omega = 1$ is definitely the most attractive possibility. It is the only value of Ω which is not changing with time in the course of the universe evolution. In the universe dominated by any normal matter all other values of Ω run away from 1. To see this let us rewrite eq.(5), using definitions (2) and (3), in the following way:

$$\Omega^{-1} = 1 - \frac{C}{\rho a^2} \quad (30)$$

where $C = 3km_{Pl}^2/8\pi = const$. For nonrelativistic matter with negligible pressure $\rho \sim 1/a^3$ and for relativistic one with the pressure $p = \rho/3$, $\rho \sim 1/a^4$. In the general case the behaviour of ρ is governed by the covariant energy conservation:

$$\dot{\rho} = -3H(\rho + p) \quad (31)$$

and for normal matter with positive pressure $\rho < 1/a^3$ when $a \rightarrow \infty$. Thus for an open universe ($C < 0$) dominated by nonrelativistic matter $\Omega \sim 1/a \rightarrow 0$ when $a \rightarrow \infty$ and for the closed universe ($C > 0$) $\Omega \rightarrow \infty$.

For vacuum energy $p = -\rho$ and so $\dot{\rho} = 0$ and $\rho a^2 \rightarrow \infty$. Correspondingly $\Omega \rightarrow 1$ with rising a . It would be very interesting if an adjustment mechanism is found which would lead to the equation of state such that $\rho a^2 = const$ which in turn would lead to Ω unchanging in the course of expansion.

Assuming that at the present day Ω is somewhere between 0.1 and 2, we see that the law (30) implies $(\Omega - 1) = O(10^{-15})$ at the nucleosynthesis epoch. At earlier stages the finetuning should be even stronger. This is the well known flatness problem which was beautifully solved by the inflationary proposal [33, 34]. Typically in most inflationary models the universe is dominated by the vacuum-like energy so that $\rho = const$ and $a \sim \exp(H_I t)$. Correspondingly $(\Omega_{infl}^{-1} - 1) \sim \exp(-2H_I \tau)(\Omega_i^{-1} - 1)$ where τ is the duration of inflationary stage, H_I is the Hubble parameter during inflation, Ω_{infl} is the value of Ω at the end of inflation, and Ω_i is

the initial (preinflationary) value of Ω . When inflation is over, Ω runs away from 1 and at the present day reaches the value:

$$\Omega_0^{-1} - 1 = (\Omega_{infl}^{-1} - 1)(z_{infl} + 1)^{-2}(\rho_{infl}/\rho_0) \approx (\Omega_{infl}^{-1} - 1)(T_R/T_0)^2 \quad (32)$$

where subzero means the present-day values, $(z_{infl} + 1) = (a_0/a_{infl}) \approx (T_R/T_0)$ is the redshift corresponding to the end of inflationary era, T_R is the temperature of the universe heating at the end of inflation, and $T_0 = 2.7 K$ is the present-day temperature of the cosmic microwave background radiation. We assumed that the energy density at the end of inflation is $\rho_{infl} = (\pi^2/30)g_*^R AT^4$ where g_*^R is the number of relativistic species at $T = T_R$ and $A > 1$ is the coefficient which accounts for the slow process of heating due to the weak inflaton coupling. The last equation in (32) was obtained under assumption $Ag_*^R(\rho_{cmb})/\rho_0 \approx 1$. The necessary duration of inflationary stage is approximately equal to $H_I\tau \approx \ln(T_R/T_0) = 52 + \ln(T_R/10^{10} GeV)$. This gives $\Omega_0^{-1} - 1 = O(1)$. Even a slightly longer inflation would result in $|\Omega_0^{-1} - 1| \ll 1$ and a slightly shorter one would be in a gross disagreement with observations. Typically inflationary models give $H_I\tau \gg 1$ which results in $|\Omega_0^{-1} - 1| \leq 1 \pm 10^{-4}$. The deviation from 1 in the r.h.s. of this relation is connected to the density inhomogeneity at the present horizon scale. So $\Omega = 1$ is considered as a robust prediction of inflationary cosmology.

Recently with accumulation of new astronomical data which may request $\Omega < 1$ there appeared models where a possibility of having $\Omega \neq 1$ in inflationary frameworks was advocated [35]. All these models request a tuning of inflationary and postinflationary stages in such a way that $\exp(2H\tau) = (T_R/T_0)^2(\Omega_0^{-1} - 1)^{-1}$. This condition looks very strange because physically these epochs are not related. The condition of small density perturbations which is not easy to realize in these models imposes an extra restriction making these models even less natural.

Thus even if $\Omega \neq 1$ is in principle possible in inflationary models the flatness problem in such models is similar to that in the old Friedman cosmology. Still it is very difficult

(if possible) to create our universe without inflationary stage and if it is indeed proven that $\Omega_{matter} < 1$ a much better solution which naturally agrees with inflation is a nonzero vacuum energy such that $\Omega_{tot} = \Omega_{matter} + \Omega_{vac} = 1$. On the other hand if we observe that $\Omega_{tot} \neq 1$ it would be a strong argument against inflation (I have to admit that this opinion is not shared by everybody).

8 Intermediate Summary

Let us summarize what is known about energy density of different forms of matter in the universe.

1. The luminous matter which is undoubtedly made of the usual particles, protons, neutrons, and electrons, contributes very little to the total mass of the universe:
 $\Omega_{lum} \approx 3.5 \times 10^{-3} h_{100}^{-1}$.
2. Primordial nucleosynthesis gives for the *total* amount of baryons $\Omega_{bar}^{(NS)} = 4 \times 10^{-3} \eta_{10} h_{100}^{-2}$ with $\eta_{10} = 1 - 7$. At the moment there is no consensus whether η is relatively high (near the upper bound) or at the lower end of the permitted interval. In the first case we should expect plenty of invisible baryons in the universe while in the second all baryons may be visible.
3. Measurements of rotational velocities in gravitationally bound systems give, depending on scale, $\Omega = 0.1 - 0.3$.
4. From the large scale flows follows $\Omega_{lsf} = [b(0.74 \pm 0.13)]^{5/3}$ with the biasing parameter $b = O(1)$.
5. From the lower bound on the universe age $t_U > 12 \text{ Gyr}$ and $h_{100} > 0.7$ follows $\Omega_{matter} < 0.2$.

6. Measurements of X-ray emission from rich galactic clusters give $\Omega_{clustered} \leq 0.15h_{100}^{-1/2}/(1+0.55h_{100}^{3/2})$.
7. Inflationary cosmology predicts $\Omega_{tot} = 1$ with a very good accuracy.

It is rather difficult to say now what is the best choice in this situation. The answer would be different depending on the personal prejudices of the author. To my mind the odds are the best for the following: $\Omega_{bar} \approx \Omega_{vis} \approx 0.5\%$; $\Omega_{DM} = 0.1 - 0.2$, and $\Omega_{vac} = 0.8 - 0.9$ with $\Omega_{tot} = 1$.

9 Possible Forms of Dark Matter

The simplest and most economical is to assume that all dark matter consists of the usual known particles: protons, neutrons, and electrons, without unnecessary introducing quantities according to Occam. Unfortunately this idea of a large amount of dark baryons contradicts the nucleosynthesis bound and the only visible way to overcome this bound is to conceal the majority of baryons in primordial black holes. It can be done but with a rather exotic model of baryogenesis which gives a very big baryon asymmetry inside relatively small bubbles [36]. When the temperature drops below the proton mass, very big density perturbations evolve which result in almost 100% collapse of the regions with high baryon asymmetry. As is argued in ref.[36] (see also [37]) if baryonic black holes have been formed sufficiently early the bound (18) may not be applicable and Ω_B close to 1 is permitted.

There are many direct astronomical restrictions on the baryonic dark matter in all possible forms which have been reviewed in lectures [38]. If the dark baryons are in the form of black holes the following limits are to be fulfilled. Black holes with $M > 10^6 M_\odot$ would lead to formation of too heavy galactic nuclei. Black holes with $M > 10^4 M_\odot$ would disrupt globular clusters. Black holes with $M < 100 M_\odot$, which have been formed in the process of heavy star evolution, are excluded because the star-progenitors would eject too much metals into interstellar medium. However primordial black holes are not subject to the last bound and for

them $M < 100M_{\odot}$ is permitted. Moreover, as we mentioned above, even the nucleosynthesis bound for them may be avoided.

Another possible candidate for dark baryons are brown dwarfs, light star-like bodies consisting of hydrogen and helium with the mass $M < 0.08M_{\odot}$, so that the reaction of hydrogen fusion is not ignited. Dark baryonic matter could also be in the form of the so called "Jupiters", planetary-type objects with $M < 10^{-3}M_{\odot}$. Compact stellar remnants: white dwarfs and neutron stars ($M = (0.4 - 2)M_{\odot}$) are also viable candidates. And at last one can expect some dark baryons to be in the form of the cold diffuse gas clouds.

During the last 3 years several groups of astronomers (EROS, MACHOS, OGLE) are looking in our galaxy for dark star- or planetary-size objects with the mass in the range $(10^{-8} - 10^3)M_{\odot}$ using the gravitational microlensing effect [39]. These objects have got the name MACHOS which is the abbreviation for Massive Astrophysical Halo Objects. When such an object happened to be on the line-of-sight between the observer and a star the brightness of the star may considerably increase due to gravitational focusing of light. The luminosity curve should have a symmetric form as a function of time and should not depend upon the light frequency. Around 100 events of this type have been observed to the present time. Unfortunately it is difficult to judge if these events are induced just by the ordinary stars or by the invisible celestial bodies which are looked for. The experiments are now in progress and more data are being accumulated. Moreover the new series of observations are now done in real time when the change of star brightness is immediately registered, while previously the analysis of the accumulated data was performed. So hopefully in the nearest future a more decisive conclusion can be made. Still even if all the observed events are induced by MACHOS it seems very plausible that they cannot account for all dark matter in the universe. For the recent review see e.g. ref.[40].

If we confine ourselves to the particles which are known to exist, the next simplest possibility for dark matter are massive neutrinos with $m = O(10\text{ eV})$. Though we cannot predict theoretically the magnitude of the neutrino mass, there is no known principle that

forbids nonzero m_ν and so it is quite natural to expect them to be massive. Unfortunately the theory of large scale structure formation excluded massive neutrinos as dominant bearers of dark matter at least in simple models with scale free spectrum of initial perturbations. It is an interesting question whether there exists a spectrum of initial perturbations which permits to describe the universe structure with neutrinos and baryons only.

If baryons and massive neutrinos cannot account for the observed Ω_{DM} there are still plenty of possibilities but now we have to turn to hypothetical not yet discovered particles or new objects or fields. The most popular now candidate for the dark matter particle is the lightest supersymmetric particle (LSP). In simple supersymmetric models this particle is stable due to conservation of R-parity, $R = (-1)^{B+L+2S}$ where B and L are respectively baryonic and leptonic charge and S is the particle spin. So defined R -parity has the value (+1) for all usual particles and (-1) for all (not yet discovered) superpartners.

In models with low energy supersymmetry the SUSY breaking scale is phenomenologically acceptable in the interval $\Lambda_{SUSY} = (0.1 - 1) TeV$. With this scale of supersymmetry breaking the theory naturally predicts the relic abundance of LSP's near $\Omega = 1$. Indeed the number density of the primordial LSP's which survived to the present epoch can be roughly estimated as [41]:

$$n_{LSP} = \frac{n_\gamma}{\sigma v m_{LSP} m_{Pl}} \quad (33)$$

where σ is the cross-section of annihilation of LSP's, v is their velocity, and $n_\gamma = 400/cm^3$ is the number density of photons in cosmic microwave background. With $\sigma v = \alpha^2/m_{LSP}^2$ where $\alpha \approx 10^{-2}$ and $m_{LSP} \approx \Lambda_{SUSY}$ we get

$$\Omega_{LSP} \approx 0.04 h_{100}^{-2} (m_{LSP}/TeV)^2 \quad (34)$$

More details about and a review on supersymmetric dark matter can be found e.g. in paper [42].

However the conservation of R-parity is not obligatory. No theoretical principle like e.g. gauge invariance demands that. So it is quite natural to expect that it is indeed broken.

Moreover if we believe the principle, which has been established by the development of the elementary particle physics during the last 30 years, that everything that can be broken should be broken, R-parity is surely broken. If this is the case the LSP should be unstable. Still depending on the mechanism and the strength of breaking the life-time of LSP's can be very long like e.g. the life-time of proton which is stable in all known experiments and observations despite an almost certain baryonic charge nonconservation. If so, the long-lived LSP can well be the dark matter particle. If however their life-time is cosmologically short, they could not contribute to dark matter and other candidates are wanted.

In models with broken R-parity an interesting candidate for dark matter particles can be massive majorons [43, 44] i.e pseudogoldstone bosons appearing due to spontaneous breaking of leptonic charge conservation. The majorons may have mass in KeV range and the number density corresponding to $\Omega = O(1)$. The universe structure formation in such model has been considered in ref.[44] and reasonably well agrees with the observed picture (see section 12).

There could exist some other stable (or cosmologically long-lived) particles that may be bearers of dark matter. They could be produced thermally, like LSP's or (massive) neutrinos in hot equilibrium primeval plasma. Depending on the strength of their interactions and the magnitude of the mass they may be relativistic when they are decoupled from the cosmic plasma (e.g. neutrinos) or nonrelativistic (e.g. LSP's). Another possible mechanism of production may be related to a phase transition and in this case even very light particles would be produced in nonrelativistic state. An example for that is the well known axion which is also considered as a serious candidate for dark matter particle. The effective potential for axion field has the form of Mexican hat after the underlying $U(1)$ -symmetry is broken at high temperature. Later on at QCD energy scale the bottom of the Mexican hat potential became tilted and because of that axions acquire a nonzero mass. At that moment coherent oscillations near the potential minimum begin. This can be understood as a production of Bose condensate of axions (at rest). Though the mass of axion is extremely small, $m_a =$

$O(10^{-5} \text{ eV})$, the axions, thus produced, remain nonrelativistic because of negligibly weak interactions with the plasma.

Some other, maybe more exotic, and because of that more interesting, candidates for dark matter are considered in the literature like e.g. topological or nontopological solitons with astronomically large size. Topological defects (or topological solitons) might be formed in the course of phase transitions accompanying spontaneous symmetry breaking when the universe expanded and cooled down. Depending upon the symmetry group different kinds of objects can be created. If the corresponding symmetry group is $U(1)$ then vortex lines (or cosmic strings) would be formed which could both make contribution into dark matter and create density inhomogeneities [45, 46]. If a discrete symmetry is broken domain walls separating different vacua are created. Domain walls with a natural (whatever it means) scale are cosmologically forbidden since they would create too large angular fluctuations of temperature of cosmic microwave background [47] but if the symmetry breaking scale is extremely low, they are allowed and can even serve as seeds for structure formation [48]. Spontaneous breaking of $SU(2)$ or $O(3)$ gives rise to production of monopoles. These three types of objects exhaust all possible topologically stable solitons in three dimensions. There could also be unstable but long-lived ones which might be cosmologically interesting. For the review see e.g. refs. [49, 50].

I think that a very interesting possibility for dark matter is a classical field which possibly compensates nonzero vacuum energy. The theory of such field has not yet been seriously considered (even at a toy model level) but the cosmological constant problem makes the existence of such a field very probable.

At the present level of our knowledge we cannot say what is the theoretically preferable candidate for dark matter particles. We only can make a judgement on the basis of the universe structure formation comparing the picture predicted by a model with a particular kind of dark matter particles and astronomical observations.

10 Structure Formation. Basic Assumptions.

The universe is believed to be homogeneous on the average at very large scales, $l > l_1 \approx 100 \text{ Mpc}$. This hypothesis is based on the observed isotropy of the cosmic microwave background and on the absence of the observed structures at $l > l_1$. However the present-day accuracy of measurements is not good enough to exclude structures at larger scales so the last statement may be questioned. At smaller scales the universe is very inhomogeneous: there are galaxies, their clusters and superclusters, large size voids, and possibly even a periodicity (or at least structures) with the characteristic wave length about 100 Mpc [51].

The task of the theory of large scale structure formation is to give a qualitative description of the distribution of matter in the universe. This is done with the following four essential assumptions of a different level of creditability:

1. It is assumed that the structure of the universe has been formed as a result of evolution of initially small fluctuations under the action of gravitational forces. This is the only solid assumption of the theory with which practically everybody agrees. Still even this sometimes is questioned and speculations about possible nongravitational processes or initially large fluctuations are put forward.

2. The next very important assumption concerns the nature and the spectrum of initial perturbations. For many years the origin of primordial fluctuations at very large scales remained mysterious. A natural idea that density inhomogeneities were generated by quantum or thermal fluctuations could not be realized in frameworks of the Friedman cosmology with the power law expansion regime, $a(t) \sim t^\alpha$, because the corresponding wave length were negligibly small in comparison with the necessary astronomical scales. This difficulty was resolved in inflationary scenario [52] where the wave lengths of perturbations were exponentially inflated, $l_0 \rightarrow l_0 \exp(H\tau)$, and what's more their amplitudes were amplified in the course of expansion. In a sense the plan was overfulfilled because the density perturbations generated in this way would be too high unless an unnaturally weak interaction of the infla-

ton field is assumed. It can be shown that for the inflaton with the self-interaction potential $U(\phi) = \lambda\phi^4$ the magnitude of density perturbations with wave length l at the moment of horizon crossing is roughly given by the expression:

$$\delta = \frac{\delta\rho}{\rho} \approx 100\sqrt{\lambda}[1 + 0.01 \ln(l/l_{gal})]^{3/2} \quad (35)$$

For the details see either the original papers [52] or the book [53]. Note that to create the necessary magnitude of fluctuations, $\delta < 10^{-4}$, bounded by the data on the angular fluctuation of the background radiation temperature, one needs $\lambda < 10^{-12}$.

A competing source of density perturbations may be astronomically large topological defects mentioned in section 9. The simplest and the most attractive possibility to my mind are cosmic strings. Unfortunately the accepted $SU(3) \times SU(2) \times U(1)$ -theory of particle interactions does not lead to cosmic strings (neither to any other mentioned above stable topological defects) and at the present level of our knowledge we cannot say if higher energy modifications of the theory definitely request existence of such objects.

If we neglect the log-dependence on the scale, the spectrum of fluctuations given by eq.(35), coincides with that proposed by Harrison [54] and Zeldovich [55] some time before inflationary cosmology was discovered. This is the so-called flat or scale-free spectrum of fluctuations. It is the simplest possible primordial spectrum which does not introduce any particular scale to the theory. With an advent of inflation this kind of spectrum got theoretical justification and now is commonly used as the basic spectrum in calculations of structure formation. Sometimes as a simple generalization an arbitrary power law spectrum is considered:

$$\delta^2 = \left(\frac{\delta\rho}{\rho}\right)^2 \sim \int \frac{dk}{k^n} \quad (36)$$

The Harrison-Zeldovich spectrum corresponds to $n = 1$. The spectrum with $n \neq 1$ may appear in some inflationary models [56] but with n not much different from unity. If we permit in principle an existence of primordial spectrum with an arbitrary n , introducing in this way a new scale to the theory, a combination of several terms with different n as well as

a more complicated function are also permitted but without a guiding principle for choosing a particular form of the spectrum, the theory would completely loose its predictive power.

3. Because of stochastic nature of the fluctuations one should consider average values and this explains in particular the consideration of δ^2 instead of δ in eq.(36). Stochastic properties of $\delta(x)$ are fixed by the following hypothesis. It is assumed that the fluctuations are gaussian with delta-correlated Fourier modes. In other words if the relative amplitude of the fluctuations is Fourier expanded:

$$\delta(x) \equiv \frac{\delta\rho(x)}{\rho} = \int \frac{d^3k}{(2\pi)^3} \bar{\delta}(\vec{k}) e^{i\vec{k}\vec{x}} \quad (37)$$

the averaged product of the mode amplitudes satisfies the condition

$$\langle \bar{\delta}(\vec{k}) \bar{\delta}(\vec{q}) \rangle = (2\pi)^3 f(\vec{k})^2 \delta(\vec{k} - \vec{q}) \quad (38)$$

(hopefully the delta-function here is not confused with the relative magnitude of density fluctuations $\delta(\vec{x})$ and its Fourier transform $\bar{\delta}(\vec{k})$).

It is also assumed that the spectrum is isotropic so that the function f is a function of the absolute value of k only, $f(\vec{k}) = f(k)$. (Note in passing that this assumption is not obligatory but we do not know any reasonable physical mechanism for the anisotropy.)

Using these equations we can easily find

$$\langle \delta^2(x) \rangle = \int d^3k f^2(k) \quad (39)$$

With $f^2 = \text{const}/k^{n+2}$ it coincides with eq.(36).

4. Except for the three hypothesis discussed above, the structure formation evidently and heavily depends upon the properties of matter which presents building blocks for the structures. In contrast to the previous three assumptions there is no consensus with respect to the universe chemical content and a plethora of different possibilities is proposed. From the point of view of structure formation all possible forms of matter particles are classified as cold, hot, and, for the intermediate case, warm. Cold matter or CDM (here "D" stands

for "dark" because it is assumed that the universe is dominated by dark matter) is formed by particles which were nonrelativistic at a rather early stage when the galactic size crossed the horizon. Hot matter (or HDM) is formed by particles which were relativistic at that moment, and correspondingly warm particles were semirelativistic. This property of dark matter particle determines the characteristic size of objects formed.

At the present day there are several models which more or less successfully describe structure formation under the first three assumptions and with different hypotheses about the matter content of the universe: CDM plus vacuum energy [57], mixed hot plus cold dark matter [58], unstable particles decaying into relativistic species plus CDM [59], HDM plus astronomically large seeds like topological defects. These models request comparable contributions of different forms of matter which are not physically related at least at the present level of our understanding and by this reason do not look natural. On the other hand we already have an example of such unnaturalness realised in the universe, namely the energy density of baryons is relatively close to that of dark matter (compare Ω_{bar} with Ω_{DM}) though they may be different by several (many!) orders of magnitude. At the moment it is not clear if this rough coincidence is an indication of an unknown deep physics, or there is something wrong in our understanding of dark matter, or we have to invoke the anthropic principle to explain that. Models with a single dominant form of matter demand a deviation from the flat spectrum of primordial density fluctuations.

It is usually assumed that the dark matter is collisionless (or in other words its selfinteraction as well as interactions with other particles may be neglected) but this is not necessarily so and the models with self-interacting dark matter are also considered [60, 61, 44].

We will briefly discuss the relevant properties of models of structure formations with different forms of dark matter particles or fields below.

11 Structure Formation. Basics of the Theory.

The evolution of small density perturbations is described by the usual hydrodynamic equations of selfgravitating perfect fluid with some complications related to the expanding cosmological background and effects of general relativity. The dynamics of liquid element is governed by the forces of gravity and pressure. If the pressure dominates then the perturbations evolve as sound waves. For the collisionless liquid like e.g. that of neutrinos the perturbations are erased by free-streaming which formally very much resembles the effect of pressure. In the case when gravity dominates, the element of liquid would be indefinitely compressing if the equation of state remains unchanged and correspondingly no new pressure forces come into play. In this case the density contrast would be rising and this results in formation of structures in the universe. Evidently with an increasing size or mass of a system gravity would ultimately dominate pressure while for small systems pressure is normally the dominant force. The boundary value of the mass and size are called respectively Jeans mass, M_J , and Jeans wave length, λ_J . They are named so after Jeans who was the first to study the phenomenon of gravitational instability at the beginning of this century.

The basic equations which describe evolution of density perturbations in the nonrelativistic perfect fluid with mass density ρ and pressure density p are the continuity equation:

$$\partial_t \rho + \vec{\partial}(\rho \vec{v}) = 0, \quad (40)$$

the Newtonian equation of motion ($m\ddot{\vec{x}} = \vec{F}$):

$$\partial_t \vec{v} + (\vec{v} \vec{\partial}) \vec{v} + \vec{\partial} p / \rho + \vec{\partial} \phi = 0, \quad (41)$$

and the Newtonian equation for the gravitational potential ϕ :

$$\Delta \phi = 4\pi \rho G_N \quad (42)$$

Perturbative expansion of these equations over homogeneous background $\rho = \rho_0 + \rho_1(t, \vec{x})$, $\vec{v} = \vec{v}_1(\vec{x}, t)$, etc, permits to derive the second order linear differential equation describing

evolution of density perturbations:

$$\ddot{\delta} - v_s^2 \Delta \delta - 4\pi \rho_0 G_N \delta = 0 \quad (43)$$

where v_s is the velocity of sound.

Making Fourier transform of this equation one sees that short wave modes are oscillating while the long wave ones are exponentially rising, $\delta \sim \exp(\sqrt{4\pi G_N \rho_0 - v_s^2 k^2} t)$. The boundary value of the wave vector $k_J^2 = 4\pi G_N \rho_0 / v_s^2$ is called the Jeans wave vector and the inverse quantity $\lambda_J = 2\pi / k_J$ is the Jeans wave length. The Jeans mass is the mass contained inside the volume bounded by λ_J and is equal to $M_J = 4\pi(\lambda_J/2)^3 \rho_0 / 3 = \pi^{5/2} v_s^3 / 6 G_N^{3/2} \rho_0^{1/2}$. In realistic cosmological situation these expressions remain true with the substitution the cosmological energy density $\rho_0(t) \sim 1/t^2$ instead of $\rho_0 = \text{const}$. Because of that the character of instability becomes different, it is no more exponential but of a power law as we have seen in sec.5.

To derive the equations of motion in Newtonian approximation with the account of the universe expansion one has to change all the derivatives to the covariant ones in the Robertson-Walker metric, to substitute as zero order approximation the cosmological time dependent energy density $\rho_0(t)$ (as we have already mentioned) and the nonvanishing zero order velocity, $\vec{v}_0 = H\vec{r}$. Otherwise the procedure is exactly the same as above. The resulting equation for the Fourier transformed amplitude of density perturbations δ_k is similar to eq. (43) with an extra "friction" term proportional to $H = \dot{a}/a$:

$$\ddot{\delta}_k + 2(\dot{a}/a)\dot{\delta}_k + (v_s^2 k^2 / a^2 - 4\pi G_N \Omega \rho_0) \delta_k = 0 \quad (44)$$

The limit of small k/a of this equation gives eq.(23). More details about the material presented above and below can be found in the books [23, 5] or in the lectures [24, 62].

It is essential for structure formation that perturbations in nonrelativistic matter do not grow in the universe dominated by a *uniformly* distributed relativistic matter. In this case the term which drives the instability in eq.(44) is changed to $4\pi \rho_0 G_N \epsilon$ where $\epsilon = \rho_m / \rho_0$, ρ_m is the

energy density of nonrelativistic matter, and ρ_0 is the dominant cosmological energy density of relativistic matter. Since by assumption $\epsilon \ll 1$, the driving force becomes negligible and rising modes are not generated. If the relativistic matter is itself inhomogeneous then there are unstable modes but usually inhomogeneities in relativistic matter are quickly erased by free streaming or diffusion. Correspondingly cosmological inhomogeneities started to grow only at matter dominated stage.

General relativistic treatment of the problem is more complicated. One has to solve the Einstein equations for the metric with corrections from inhomogeneous fluctuations. There is an ambiguity related to the freedom in choice of different coordinate systems (the so called gauge freedom), so that one has to distinguish between the gauge artifacts and real physical effects. This analysis was first done by Lifshits in 1946[63]. The gauge independent formalism was developed by Bardeen [64] relatively recently. In many cases however it is sufficient to use the Newtonian approximation for qualitative (and often quantitative) understanding of the phenomena.

While the analysis of the evolution of small perturbations is relatively simple the non-perturbative regime which started when $\delta = O(1)$ is much more complicated. No reliable analytical or semianalytical methods have yet been developed and the calculations are made by numerical simulations with a large number of noninteracting (except for gravity) particles. By necessity many essential properties of the system are neglected but nevertheless one may hope that this approach gives a reasonable description of the structure formation at the stage of large perturbations.

12 Evolution of Perturbations with Different forms of Matter.

Let us first consider density perturbations in the cosmological gas of light neutrinos with mass in 10 eV range. At an early stage when neutrinos are relativistic density contrasts

are smoothed down by their free motion. The perturbations could start rising only when neutrino momentum is redshifted down to nonrelativistic values. The characteristic size of the fluctuations which survived the erasure by free streaming, is given by the path which neutrinos could travel till they became nonrelativistic, $l_m \approx 2t_m$. Here t_m is the universe age at that moment. It is assumed that neutrinos moved with the speed of light and the coefficient 2 is connected with the universe expansion (in the static universe the path would be just $l = t$). We find t_m using equations (14,15): $t_m = 4 \times 10^{20} \text{ cm}(10 \text{ eV}/T_m)^2$. We choose $T_m = m_\nu/3$ because the average neutrino energy in thermal equilibrium state is $\langle E \rangle \approx 3T$. To calculate the magnitude of l_m at the present day we have to multiply it by the red shift $z_m + 1 = T_m/T_\nu^0$ where $T_\nu^0 = 1.93 \text{ K}$ is the present day temperature of the cosmic massless neutrinos. Thus we get

$$l_m^{(0)} = 16 \text{ Mpc}(10 \text{ eV}/T_m) = 48 \text{ Mpc}(10 \text{ eV}/m_\nu) \quad (45)$$

The mass of matter inside the radius $l_m/2$ is $M_m \approx 3 \times 10^{15} M_\odot (10 \text{ eV}/m_\nu)^2$. It is 3-4 orders of magnitude bigger than galactic mass. Note that $l_m^{(0)}$ and M_m coincide correspondingly with λ_J and M_J given above if we formally take the speed of sound equal to the speed of light. This is indeed true for relativistic gas. Eq.(45) gives the mass of the smallest size objects which could be initially formed in neutrino dominated universe. Smaller objects is difficult to create and this was the reason why a beautiful idea of neutrino universe was abandoned. Particles with a larger mass like e.g. $m = 300 \text{ eV}$ would create structures with characteristic size close to that of galaxies. As we mentioned above these are particles of warm dark matter. There is a renewed interest to these particles now. Unfortunately no particles with such mass are observed experimentally but there are some theoretical models (see e.g. refs.[43, 44]) where such particles may exist and contribute to dark matter.

During several last years, before the COBE measurements [65] of the large angle anisotropy of the cosmic microwave background radiation, the model with cold dark matter was definitely the most popular (for the review see e.g. refs. [58, 66]). Based on a very simple

assumption of scale-free spectrum of primordial fluctuations (Harrison-Zeldovich spectrum), the model reasonably well described the galaxy distribution at the scale of tens megaparsec with the only free parameter, the overall normalization of the spectrum. However the COBE measurements have fixed the normalization at large scale end of the spectrum and with this normalization the prediction for smaller scales became a factor two above the observations. An evident possible cure is to change the shape of the spectrum of primordial fluctuations. It seems to be a very interesting possibility but since this idea is not supported by the inflationary scenario, the major line of investigation is the consideration of different forms of dark matter with the same flat spectrum of density perturbations or maybe an introduction of the cosmological constant. The basic idea of all these models was to suppress the power of the evolved spectrum at smaller scales relative to that at larger (COBE) scales.

One possibility is to shift the epoch of matter dominance (MD) to a later stage in a simple cold dark matter model. Since the characteristic scale at which perturbations started to rise in this model is determined by the horizon size at the onset of the MD stage, shifting it to a later moment gives less time for rising of the fluctuations and correspondingly less power at galactic and cluster scales. This goal can be achieved if one assumed that universe is open so that $h_{100}^2 \Omega \approx 0.2$. This model is disfavoured by inflationary scenarios which (at least in simple versions) predicts $\Omega = 1$. One can recover this prediction of inflation in the universe with low matter density if the cosmological constant Λ (or in other words, vacuum energy) is nonzero. As we have mentioned in sec. 2, the recent data indicating a rather high value of $h_{100} \approx 0.8$ support the idea of nonzero Λ with the fraction of the vacuum energy $\Omega_{vac} = 0.8$. The models with nonzero Λ give a satisfactory description of the observed structure [57] with flat spectrum and COBE normalization.

A mixed (hot+cold) dark matter scenario can also do the necessary job of diminishing the power at small scales because (initially flat) perturbations in hot dark matter are erased at scales smaller than $\sim 10^{14} M_{\odot}$ by free streaming if the dark matter is collisionless as is the case of neutrinos. A good description of the structure requests 70% of CDM and 30% of

HDM [58]. It gives even better description if there are two equal mass neutrino species each with the mass 2.5 eV [67] as is suggested by the recent indications of neutrino oscillations by the Los Alamos group.

Recently there appeared a renewed interest to the idea of structure formation with unstable particles [59]. It is assumed in these models that there exists a massive long-lived particle, usually tau-neutrino with the mass in MeV range which decayed into massless species at the epoch when the mass density of the parent particles dominated the energy density of the universe. Correspondingly the present-day energy density of relativistic particles would be bigger than in the standard scenario and the onset of MD stage would take place later.

The common shortcoming of these models is that they all demand a certain amount of fine-tuning. Generally one would not expect that the contribution from hot and cold dark matter into the universe mass are about the same, they may differ by many orders of magnitude. One would also suspect that the vacuum energy which remains constant in the course of the universe expansion is by no means related to the critical energy now (which goes down with time as m_{Pl}^2/t^2)². The models with unstable particles mentioned above are also based on the assumption of two independent components: the massive unstable particles themselves and unrelated cold dark matter. This is definitely unnatural and this shortcoming stimulated search for other models. Recently in ref. [69] a return to the model with a single cold dark matter component universe (of course except for baryons) was advocated. For successful description of the observed structure the authors need the power index of the spectrum $n = 0.8 - 0.9$, a low value of the Hubble constant, $h_{100} = 0.45 - 0.5$ (to be compatible with large Ω), and a large contribution of tensor perturbations (gravitational waves) into quadrupole fluctuation of the background radiation temperature measured by COBE, $C_T/C_S = 0.7$.

Another attempt to overcome unnaturalness of multicomponent dark matter model has

²Adjustment mechanism [10, 11] (for more references see reviews [8, 9] and the recent paper [68]) however generically gives the noncompensated amount of vacuum energy of exactly necessary order m_{Pl}^2/t^2 and so the models with vacuum energy may be more "equal than others".

been done in paper [44]. In this paper a model is considered in which unstable particles and the particles of cold (or possibly warm) dark matter are closely connected. In fact the decay of the former produces particles of the present-day dark matter. A necessary background model of this kind in particle physics was proposed some time ago [70, 71] as an attempt to find a phenomenologically acceptable description of R-parity breaking. The underlying mechanism is the spontaneous breaking of leptonic charge conservation at electroweak scale. The model contains a Majorana type tau-neutrino with the mass around MeV which decays into massive but light Majoron J with mass in KeV region. Life-time with respect to this decay, as estimated in refs. [70], could be of order days or years depending on the values of parameters, i.e. just in the interesting for us interval. It is worth noting that there is no stable SUSY particle in this model so the dark matter cannot be associated with it but the model itself produce a candidate for dark matter, namely a massive Majoron. The model possesses some features like sufficiently large diagonal coupling of ν_τ to majorons or selfinteraction of majorons which rather naturally permit to resolve appearing cosmological problems in particular the problem of extra massless particle species during primordial nucleosynthesis. The cosmological properties of this model are rather unusual and are interesting by themselves. Dark matter particles (majorons) in this model are strongly self-interacting and thus the structure formation in this model is different from the traditional one with collisionless dark matter particles [60, 61, 72, 44]. In particular the shape of galactic halo would depend on the dark matter selfinteraction. The recent data [73] might give an indication that collisionless dark matter gives a poor description of the shape of halo in dwarf galaxies.

13 Conclusion.

It is difficult to write a conclusion on something which is so evasive as dark matter. Though it is practically certain that dark matter exists, it is unknown what it is and even if there are one or more forms of dark matter. So at the present level of our knowledge we can only

make bets which are very little constrained by the poor available data.

One of the most important questions is whether Ω is indeed equal to 1 as predicted by inflation or it is considerably smaller. In the last case the universe age problem is not so severe with a large Hubble constant but still if $h_{100} > 0.75$ and $t_U > 14 \text{ Gyr}$ even $\Omega = 0$ does not help much. It is very important to know H more accurately since its value may give a weighty argument in favor of nonzero cosmological constant Λ . With nonzero cosmological constant Ω_{matter} could be relatively small (while $\Omega_{tot} = \Omega_{matter} + \Omega_{vac} = 1$) and the universe age problem does not exist. Structure formation in models with $\Lambda \neq 0$ looks reasonable too and the unnaturally close values of ρ_c and ρ_{vac} may be understood with an (unknown) adjustment mechanism. So one of the most important problems is to find a workable model for the latter or in more general terms to solve the cosmological constant mystery. If it is indeed solved by an adjustment mechanism one should expect that $\delta\rho_{vac} \approx \rho_c$ during all the history of the universe. Hence the cosmology with adjustment of vacuum energy may give somewhat different predictions for light element abundance, structure formation, etc. Probably the baryonic crisis is also resolved by a nonzero cosmological constant.

Speaking about more traditional forms of dark matter we have to note that nonzero ρ_{vac} is not sufficient to solve all problems with dark matter and some amount of "normal" dark matter is necessary. What is it is a big question. It may be LSP's or axions, or massive neutrinos. Though the latter are disfavored by the theory of structure formation, it may be premature to exclude them completely as a dominant dark matter (together with vacuum energy). If R-parity is not conserved LSP's are probably excluded but a massive majoron in this case becomes a very interesting candidate. Accelerator experiments in search of supersymmetry are very important for resolution of this problem.

Invisible baryons may also exist and to this end a clarification of situation with primordial nucleosynthesis is very desirable especially a resolution of the ambiguity with the observation of primordial deuterium. In particular it may settle down the problem with the number of massless particle species present at nucleosynthesis and exclude or confirm a hypothesis of

MeV-tau-neutrino.

In a whole we can expect a really exciting development in the nearest future.

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